if m is an integer (ν is an arbitrary constant). Substituting Eq. (2.25) into Eq. (2.24) and equating the terms with like powers of $\exp(-i\nu t)$, we obtain an infinite system of linear differential equations which must be solved successively to determine the functions $M_i(x)$.

e) Example. To illustrate the problem examined above we shall present a numerical example. We take the experimental values from [1]. The following starting data are provided to the program: $t = (0\text{-}15)$ sec; $E = 4\cdot10^9$ kg/m·sec^2; $G = 2500$ kg; ($B_0 = 84\cdot10^{-8}$, $B_\infty = 5.6\cdot10^{-8}$) $(\text{kg/m}^2\cdot\text{sec}^2)^{-m}\text{sec}^{-1}$; $\mu = 3\cdot10^{-2}$ sec^{-1}; $m = 1.72$; $h = 0.25$ m; $\rho^* = 10^3$ kg/m^3; $g = 9.81$ m/sec^2.

Figures 1 and 2 show the curves of the approximation for the bending moment M and the approximations for the deflection v as a function of the coordinate x for a fixed time t = 15 sec. The curves 1 correspond to the zeroth approximation of the problem (classical instantaneous elastic solution) and the curves 2 are the first approximation of the problem. These curves were calculated using the formulas obtained analytically (the points a and b above). It was established numerically that for the time t = 15 sec five approximations of the problem are adequate (curve 3). As the time increases more terms must be retained in the series expansion (2.7).

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FLOW AND SEPARATION OF A RAREFIED BINARY GAS MIXTURE IN A CYLINDRICAL GAP WITH SUPERSONIC ROTATION OF THE OUTER CYLINDER

V. D. Borisevich and S. V. Yupatov

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Study of cylindrical Couette flow at Knudsen numbers $Kn = 10^{-2}\text{-}1$ is not only a classical problem of rarefied gas dynamics, but is also of practical interest [1, 2]. In the case where the outer cylinder is fixed while the inner one rotates with a velocity equivalent to a Mach number $M \le 1$, the flow has been studied experimentally [3] and numerically, both by direct statistical modeling [1], and by solution of model kinetic equations [2, 4]. At supersonic inner cylinder velocities [5, 6] analyzed the effect on flow characteristics of the Mach number and gap size. Significanty fewer studies exist for the case where the outer cylinder rotates while the inner is at rest. A numerical solution of the Boltzmann equation was found for that problem in [2] for the BGK model. Flow of a rarefied binary gas mixture in a planar gap was studied for various ratios of component masses and concentrations in [7], which obtained velocity distributions and components of the viscous stress tensor in the gap.

1. The present study will perform a numerical investigation of the flow of a rarefied binary gas mixture with molecular masses $\mu_1 = 300$ and $\mu_2 = 400$ in the gap between coaxial cylinders using the direct statistical modeling method of [8]. The outer cylinder of radius r_2 rotates with an angular velocity ω_2 ($M = \omega_2 r_2/(\gamma RT_0/\mu_2)^{1/2} = 3$), and the indices 1 and 2 below will refer to quantities defined on the surfaces of the inner and outer cylinders, respectively; T_0 is the temperature of both cylinders; R is the ideal gas constant; γ is the adiabatic index for the heavy gas, equal to unity; $Kn = \lambda_2/(r_2 - r_1)$ was varied over the

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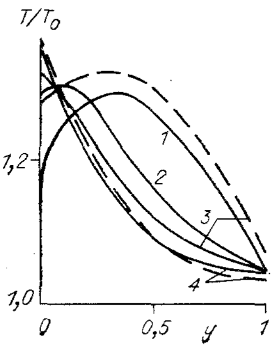

Fig. 1

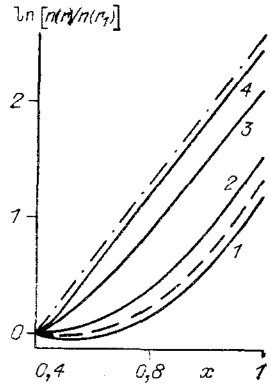

Fig. 2

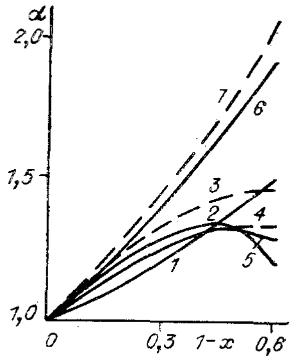

Fig. 3

range 10^{-2}-2. The "rigid sphere" model was chosen for the intermolecular interaction, with identical interaction section for the model particles. The gas mixture density at the outer cylinder surface was evaluated from the exponential distribution

$$n(r) = k \exp A(r/r_2)^2, \tag{1}$$

where $A = [c\mu_2 + (1 - c)\mu_1]\omega_2{}^2 r_2{}^2/2RT_0$, c is the relative concentration of the heavy component in the mixture. The constant k was determined by the normalization condition

$$N = 2k \int_{r_1}^{r_2} \exp A(r/r_2)^2 \, r dr.$$

The total number of model particles N used in the calculation was 8000. The algorithm was constructed such that the flow field was divided into 20 cells over radius for $Kn \geq 10^{-1}$ and into 40-50 cells for $Kn < 10^{-1}$. The characteristic time of gas temperature change t_0 was evaluated from the thermal diffusivity equation $t_0 \sim (r_2 - r_1)^2/\chi \sim \tau/Kn^2$ (where χ is the thermal diffusivity coefficient and τ is the particle mean free path time). Calculations showed that in the interval from $2t_0$ to $3t_0$ the maximum change in temperature with time is not more than 1%. Consequently, for times $t \geq 3t_0$ it can be assumed that the flow characteristics have reached a steady state with a systematic error not exceeding 1%. The volume of the statistical sample over cells after attainment of the steady state comprised $(1\text{-}5)\cdot 10^4$ particles of each component. The statistical error in determining flow characteristics was then also not more than 1%. For greatly differing component concentrations weight factors were used in the calculations as in [8].

2. The numerical calculations provided distributions of temperature, density, and the azimuthal velocity component across the gap for each component individually, as well as for the mixture as a whole. The values found for $\mu_1 = \mu_2$ were compared with the results of [2]. Figure 1 shows the gas temperature distribution across the gap ($y = (r - r_1)/(r_2 - r_1)$) for various Kn. Curves 1-4 correspond to $Kn = 10^{-2}$, 10^{-1}, $5\cdot 10^{-1}$, 0.9. The solid lines are the results of the present study, while dashes indicate data from [2]. The satisfactory agreement obtained at Kn = 0.9 is lost with increase in gas density. The divergence at low Kn is apparently related to the simplified form of the collision integral in the BGK model, which introduces an error which increases with increase in particle collision frequency.

The distribution of the log of relative gas density over the gap in $\ln n(r)/n(r_1)$ is illustrated in Fig. 2 ($x = (r/r_2)^2$). In contrast to the case where the inner cylinder is absent, the gas density distribution is dependent on Kn (curves 1-4 correspond to $Kn = 10^{-2}$, $2.5\cdot 10^{-2}$, 10^{-1}, 1), while the increase in Kn, as defined in [2], n(r) tends to an exponential Boltzmann distribution (dash–dot curve). With decrease in gas rarefaction the dependences found here approach the solution for a continuous medium and isothermal flow (dashed line). A characteristic feature of the gas density dependences obtained herein at low Kn is the presence of a minimum at the surface of the inner cylinder, which is related to a maximum in the gas temperature in this region. It follows from analysis of the equations for a continuous medium that at low Kn gas heating is related basically to the process of viscous dissipation of the gas kinetic energy. For a planar gap an analytical solution can be obtained for the temperature profile, which is parabolic in form, and symmetric about the midpoint of the gap. Meanwhile, the value of the maximum is proportional to M^2. In a cylindrical gap the dependence on M^2 is maintained, but the position of the temperature maximum is toward the inner cylinder. A similar asymmetry has been observed in the problem of rotation of the inner cylinder [5, 6].

3. The dependence of gas density distribution across the gap on Kn leads to an increase in the component separation coefficient $\alpha_0 = \frac{c_2}{1-c_2} \Big/ \frac{c_1}{1-c_1}$ with increase in rarefaction (c is the relative concentration of the heavy component at the cylinder surfaces). Thus, for a 50% mixture α_0 varies from 1.43 at Kn = 10^{-2} to 1.94 at Kn = 2. For an exponential density distribution across the gap α_0 = 2.05.

Figure 3 shows dependence of the separation coefficient $\alpha(x) = \frac{c_2}{1-c_2} \Big/ \frac{c(x)}{1-c(x)}$ on coordinate x for various Kn. The presence of a minimum in the gas density at the surface of the nonmoving cylinder at Kn = 10^{-2} leads to the development of a maximum in $\alpha(x)$ in the same region. As was shown above, the value of the temperature maximum (density minimum) depends on the Mach number for the mixture as a whole, M_m, which changes with change in mixture composition. This in turn leads to a dependence of the mixture separation coefficient on component concentration. Calculations show that at c = 90% and c = 10% the behavior of $\alpha(x)$ differs significantly (curves 1 and 2 of Fig. 3). For a 10% mixture (M_m = 2.64) the change in $\alpha(x)$ is monotonic. For a 90% mixture (M_m = 2.96) there is a maximum in $\alpha(x)$, while α_0 is 12% less than for the 10% heavy component concentration, although the pressure drop across the gap is larger in this case. The dashed line 3 of Fig. 3 corresponds to the solution for a 50% mixture, found from the equation for a continuous medium in the isothermal case, while curve 4 is the same solution, but with consideration of the temperature distribution found in the calculations. The latter agrees well with the results of the present study, with the exception of a small region near the surface of the inner cylinder (curve 5). The results indicate that in cylindrical Couette flow at low Kn, aside from the pressure gradient, the temperature distribution, which is dependent on mixture concentration, exerts a significant effect on separation.

With increase in Kn the dependence of $\alpha(x)$ on mixture composition weakens, practically disappearing at Kn = 1. For a severely rarefied gas $\alpha(x)$ is described well analytically by distribution (1) with an error not exceeding 5%, as illustrated by curve 6 of Fig. 3, calculated for Kn = 1 for 90% and 10% mixtures, and dashed line 7, obtained from density distribution (1). This can be explained by the fact that in a highly rarefied gas particle interaction is absent, as a result of which the mixture components behave independently.

The study performed shows that in a cylindrical gap flow of a binary gas mixture is described well by an exponential law at Kn ≥ 1. With increase in gas density a deviation from Boltzmann distribution occurs, which leads to dependence of the separation coefficient on mixture composition.

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